A NOVEL PARALLEL RUNNING SYSTEM OF FOUR OSCILLATORS COUPLED THROUGH AN EIGHT-PORT HYBRID

Isao OHTA and Masato HIGASHIO

Faculty of Engineering, Himeji Institute of Technology,
2167 Shosha, Himeji-shi, 671-22 Japan

Abstract

This paper treats a novel parallel running system of four oscillators equally coupled to one another through an eight-port hybrid. This system is marked by easy analyzability and adjustability from the symmetrical construction. In addition, a combined power is distinguishably delivered to an arbitrary port of four output ports, that is, can be switched into four ways. Experimental corroboration is presented also.

Introduction

Parallel running of two or more microwave oscillators has been investigated from a practical interest in power combining and a theoretical interest in the phenomenon of mutual synchronization of multiple oscillators. The parallel running structures are separated into two typical ones. One is a 2^N-oscillator system coupled through 2^N-1 3 dB hybrids such as a magic T [1], [2], and the other an N-oscillator system serially connected by N-1 directional couplers with coupling factors of 3 dB, 4.78 dB, and so forth [3].

In this paper, we treat a novel parallel running system of four oscillators using an eight-port hybrid. This system is easy to analyze and adjust, since the four oscillators stand on an equal footing with one another in the coupling circuit system. Another point of the present system is to be able to deliver a combined power into an arbitrary port of the four ports without oscillators by adjusting the circuit parameters connected to the ports. This property suggests an application to a power combining system capable of four-way switching.

We first outline the circuit construction and the underlying principle. Next, four synchronized steady-state solutions for the oscillators system are obtained. Moreover, we derive circuit conditions to stabilize the four synchronization modes by inspecting the variational equations with regard to each steady-state solution. Finally, experimental verification for power combining is presented.

Construction and steady-state solutions

Fig. 1 shows a power combining system to be considered. In the present paper, we employ an eight-port circuit consisting of a ring of four magic T's connected with each other by E-arms and by H-arms as a power combiner. Although a magic T type combiner is chosen here, other types of eight-port hybrids (comparators) [4]∿[8] also are applicable to this power combining system. Giving numbers of $1\sim4$ to the four ports having the oscillators and of $5\sim8$ to the four ports terminated in variable reflectors (loads) with reflection coefficients of $\Gamma_1\sim\Gamma_4$ as illustrated in Fig. 1, and properly choosing a reference plane at each port, we have the following scattering matrix for the eight-port network:

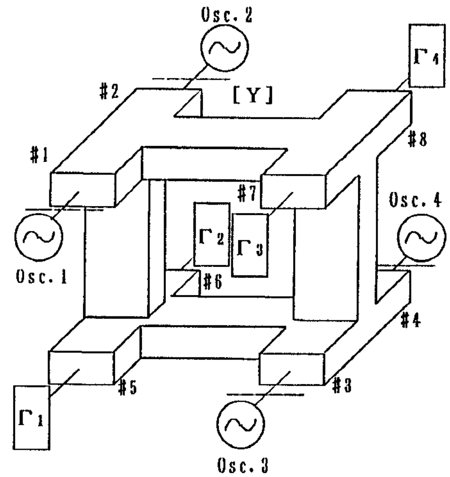

Fig. 1. Block diagram of a newly proposed parallel running system of four oscillators.

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & \boldsymbol{M} \\ \boldsymbol{M} & 0 \end{bmatrix}, \qquad (1)$$

where

$$\boldsymbol{M} = \begin{bmatrix} 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \end{bmatrix}.$$

This equation implies that the four active (oscillator) ports and the four passive (load) ports are isolated from one another severally and input

OF1

power at a port is divided into four equal parts among four ports belonging to the other group in the phase relation given by (1). Therefore, this eight-port power combiner is a variety of eight-port hybrids [7].

Now, assuming that $[Y]$ indicates an admittance matrix looking toward the coupling circuit inclusive of the four valuable reflectors (or loads) at the above reference planes of the four oscillators, we obtain the equation of synchronized oscillation

$$([Y] + [Y_G])\boldsymbol{v} = 0 \tag{2}$$

where $\boldsymbol{v}$ is the voltage vector with its components given by the RF voltage at respective reference planes, and

$$[Y_G] = \mathrm{diag}[Y_{G1}, Y_{G2}, Y_{G3}, Y_{G4}]. \tag{3}$$

To complete the list of definitions we represent the admittance of the pth oscillator as

$$Y_{Gp} = G_{ep}(V_p) + jQ_{ep}(\omega - \omega_{0p})/\omega_0 \tag{4}$$
$$p = 1,2,3,4$$

where $G_{ep}(V_p)$ is the negative conductance, V_p the amplitude of the oscillation voltage, ω_{0p} the free-running angular frequency, Q_{ep} the external Q, and ω_0 the average value of the four free-running angular frequencies.

Next, let it be assumed that each oscillator has the same parameters including the free-running frequency for simplifying the present problem. Then, all the oscillators see the same admittance owing to the symmetry of the coupling circuit and the identity of the four oscillators, and we have $V_1=\cdots=V_4(\equiv V)$, i.e., $Y_{G1}=\cdots=Y_{G4}(\equiv Y_G)$. As a result, since (2) becomes the eigenvalue problem of $[Y]$, we can find that the system falls into one of the following four synchronized steady-state solutions:

$$Y_G + y_i = 0, \quad \boldsymbol{v} = 2V\boldsymbol{u}_i, \quad i = 1,2,3,4 \tag{5}$$

where $\boldsymbol{u}_1=(1,-1,1,1)^t/2$, $\boldsymbol{u}_2=(-1,1,1,1)^t/2$, $\boldsymbol{u}_3=(1,1,1,-1)^t/2$ and $\boldsymbol{u}_4=(1,1,-1,1)^t/2$ are the eigenvectors of $[Y]$. The corresponding eigenvalues y_i is given by

$$y_i = (1 - \Gamma_i)/(1 + \Gamma_i) \tag{6}$$
$$i = 1,2,3,4$$

It is found from (5) that each oscillator in synchronism behaves as if it were terminated by the same admittance as the corresponding eigenvalue (eigenadmittance).

Condition for stability

In this section, we first derive variational equations fundamental to treating the stability of the four synchronized steady-state solutions. Now, assuming that the amplitude and the phase of the oscillation voltage of the pth oscillator deviate by slight values of δV_p and $\delta\theta_p$, respectively, from the ith steady-state solution for one reason or another, we can rewrite the voltage vector with the first-order approximation as follows:

$$\boldsymbol{v} = \boldsymbol{v}_{is} + \delta\boldsymbol{V}_i + jV\delta\boldsymbol{\theta}_i \tag{7}$$

where $\boldsymbol{v}_{is} = 2V\boldsymbol{u}_i$ and

$$\delta\boldsymbol{V}_i = \mathrm{diag}[e^{j\theta_1}, e^{j\theta_2}, e^{j\theta_3}, e^{j\theta_4}]\delta\boldsymbol{V} \tag{8a}$$

$$\delta\boldsymbol{\theta}_i = \mathrm{diag}[e^{j\theta_1}, e^{j\theta_2}, e^{j\theta_3}, e^{j\theta_4}]\delta\boldsymbol{\theta}. \tag{8b}$$

In the above equations, θ_p represents the oscillation phase of the pth oscillator in the ith synchronism, and it takes either of the two values of 0 and π as can be seen from (5). In addition,

$$\delta\boldsymbol{V} = (\delta V_1, \delta V_2, \delta V_3, \delta V_4)^t \tag{9a}$$

$$\delta\boldsymbol{\theta} = (\delta\theta_1, \delta\theta_2, \delta\theta_3, \delta\theta_4)^t. \tag{9b}$$

With consideration of the fact that the term $j\omega$ in an admittance operated on v_p indicates dv_p/dt in a dynamical sense, substituting (7) into (2) and approximating in first order with respect to small variations give

$$([Y] + Y_G[U])(\boldsymbol{v}_{is} + \delta\boldsymbol{V}_i + jV\delta\boldsymbol{\theta}_i) + s\delta\boldsymbol{V}_i + d\delta\boldsymbol{V}_i/d\tau + jVd\delta\boldsymbol{\theta}_i/d\tau = 0 \tag{10}$$

where $s=(dG_e/dV)_V V$, $[U]$ is the unit matrix, and $\tau=(\omega_0/2Q_e)t$. Moreover, it is assumed that the coupling circuit is free of frequency dependence.

Substituting the ith steady-state equation, $([Y]+Y_G[U])\cdot\boldsymbol{v}_{is}=0$ and the admittance expression (5) into (10), and using the spectral representation of $[Y]$ and the orthonormal conditions between $\boldsymbol{u}_k$'s, we can derive the following variational equations around the ith steady-state solution:

$$\tilde{\boldsymbol{u}}_k\cdot\{(y_k - y_i)(\delta\boldsymbol{V}_i + jV\delta\boldsymbol{\theta}_i) + s\delta\boldsymbol{V}_i + d\delta\boldsymbol{V}_i/d\tau + jVd\delta\boldsymbol{\theta}_i/d\tau\} = 0 \tag{11}$$
$$k = 1,2,3,4$$

where $\tilde{\boldsymbol{u}}_k$ is the adjoint vector of $\boldsymbol{u}_k$.

Next, let us examine the stability conditions for the ith mode of synchronization using (11). Considering that $\boldsymbol{u}_k$, $\delta\boldsymbol{V}_i$ and $\delta\boldsymbol{\theta}_i$ are real for all k and i, we can rewrite (11) as

1) $k = i$

$$d(\tilde{\boldsymbol{u}}_i\cdot\delta\boldsymbol{V}_i)/d\tau + s(\tilde{\boldsymbol{u}}_i\cdot\delta\boldsymbol{V}_i) = 0 \tag{12a}$$

$$d(\tilde{\boldsymbol{u}}_i\cdot\delta\boldsymbol{\theta}_i)/d\tau = 0 \tag{12b}$$

2) $k \neq i$

$$d(\tilde{\boldsymbol{u}}_k\cdot\delta\boldsymbol{V}_i)/d\tau + \{s + \mathrm{Re}(y_k - y_i)\}(\tilde{\boldsymbol{u}}_k\cdot\delta\boldsymbol{V}_i) - \mathrm{Im}(y_k - y_i)V(\tilde{\boldsymbol{u}}_k\cdot\delta\boldsymbol{\theta}_i) = 0 \tag{13a}$$

$$Vd(\tilde{\boldsymbol{u}}_k\cdot\delta\boldsymbol{\theta}_i)/d\tau + \mathrm{Re}(y_k - y_i)V(\tilde{\boldsymbol{u}}_k\cdot\delta\boldsymbol{\theta}_i) + \mathrm{Im}(y_k - y_i)(\tilde{\boldsymbol{u}}_k\cdot\delta\boldsymbol{V}_i) = 0 \tag{13b}$$

In order that the ith synchronization is stable, all $\boldsymbol{u}_k$ components of $\delta\boldsymbol{V}_i$ and $\delta\boldsymbol{\theta}_i$ have to decrease with time. For the $\boldsymbol{u}_i$ component of $\delta\boldsymbol{V}_i$, the inequality

$$s > 0 \tag{14}$$

is required from (12a), while no condition for the $\boldsymbol{u}_i$ component of $\delta\boldsymbol{\theta}_i$ is required from (12b). Since an ordinary self-excited oscillator has $s\simeq 2$, the above inequality is satisfied generally. For the other $\boldsymbol{u}_k$ components of $\delta \boldsymbol{V}_i$ and $\delta\theta_i$, the two roots of the following characteristic equation derived from (13) must have negative real parts:

$$\begin{vmatrix} s+\mathrm{Re}(y_k-y_i)+\lambda & -\mathrm{Im}(y_k-y_i)\cdot V \\ \mathrm{Im}(y_k-y_i)/V & \mathrm{Re}(y_k-y_i)+\lambda \end{vmatrix} = 0 \qquad (15)$$

To satisfy the above requirement, it is necessary and sufficient that

$$s + 2\mathrm{Re}(y_k - y_i) > 0 \qquad (16a)$$

$$s\mathrm{Re}(y_k - y_i) + \{\mathrm{Re}(y_k - y_i)\}^2 + \{\mathrm{Im}(y_k - y_i)\}^2 > 0 \qquad (16b)$$

hold.

Now, we consider the system in which the powers of the four oscillators in the first synchronism (i=1) are combined into a matched load. In this case we may choose that y_1=1 (or Γ_1=0) and, for simplicity, assume that all the other admittances have a same value of y_k. Then, a region

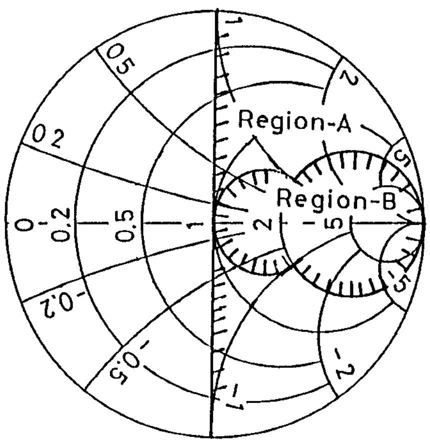

Fig. 2. Stability region of the first synchronization (region A) and instability region of the other synchronizations (region B) on a Smith admittance chart for y_k(=y_2=y_3=y_4) when the output port (#5) is matched (y_1=1 or Γ_1=0).

where y_k satisfies (16), in other words a stability region of the first mode for an admittance of the other passive ports, can be drawn as region A on the Smith admittance chart in Fig. 2 with assumption of s=2. On the other hand, if we replace y_k and y_i in (16) by y_1=1 and y_k, respectively, and analyze a region where (16) is unsatisfied, we obtain region B in the same figure. This region is an instability region of the other modes for y_k. Thus, it is found that by fixation of y_k (Γ_k) in the common region between both regions (i.e., region B), only the first mode can be realized with the desired stability. The above discussion can be applied to other systems with different output ports by renumbering the admittances, y_k and y_i appropriately for the desired mode.

Experimental results

A power combining system shown in Fig. 1 was constructed of four Gunn oscillators, whose output powers and free-running frequencies were equally adjusted to 62.4 mW and 9.2 GHz, respectively, and whose external Q's were about 78 on the average. Variable reflectors or loads was composed of a sliding screw tuner and a matched load inclusive of measuring equipments such as a power sensor, a frequency counter and a spectrum analyzer as illustrated in Fig. 3. The four reflection coefficients were adjusted to -0.43 by an inserted depth and a position of the screw of the tuner. Then, adjusting some one of the reflection coefficients to zero (i. e., bringing the depth of the screw of

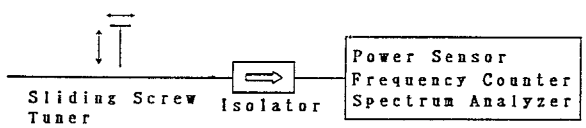

Fig. 3. Block diagram of variable reflectors (or loads) composed for measurements of combined output power, synchronizing frequency and output spectrum.

Table 1
Measured Output Powers (Ratios to the Total Power) into the Four Passive Ports and Synchronizing Frequencies in the Four Synchronizations

Output Port Number	1st Mode	2nd Mode	3rd Mode	4th Mode
5	233.1mW (93.4%)	4.7mW (1.9%)	0.34mW (0.14%)	0.26mW (0.10%)
6	2.6mW (1.0%)	235.6mW (94.4%)	0.25mW (0.10%)	0.13mW (0.05%)
7	3.7mW (1.5%)	1.3mW (0.51%)	242.8mW (97.3%)	5.4mW (2.2%)
8	1.1mW (0.44%)	1.5mW (0.59%)	2.1mW (0.82%)	241.6mW (96.8%)
Frequency	9196.3MHz	9195.4MHz	9199.9MHz	9201.2MHz

some tuner to zero), we produced a combined power of the four oscillators to the matched port. This fact tells us that the system can switch a combined power to four ways. Table 1 gives measured output powers emerging from the four ports and synchronization frequencies in each synchronism. The measured powers are converted into the values at the input port of each isolator. The power-combining efficiencies at the output ports are 93.4 ∿ 97.3% and the leaky powers from the off-ports are 0.05 ∿ 2.2% of the total power of the four oscillators. Moreover, frequencies in the synchronism are approximately equal to that in free-running.

Conclusions

We have made both a theoretical and experimental investigation of a novel power combining system of four oscillators symmetrically coupled through an eight-port hybrid. This system is characterized by easy analyzability and adjustability resulting from the equal footing of the four oscillators and the four loads in the system. In addition, it has been shown that output ports can be switched simply, e.g., by putting some metallic posts (or screws) into and out of the waveguides of the four passive ports.

An application to high-power oscillators system and the development of a multiport hybrid capable of connecting more oscillators would be interesting subjects for further work.

Acknowledgment

This work was supported in part by a Grant-in-Aid for Science Research from the Ministry of Education, Japan.

References

[1]H. Fukui, "Frequency locking and modulation of microwave silicon avalanche diode oscillator," *Proc. IEEE*, vol. 54, pp.1475-1477, Oct. 1966.
[2]S. Mizushina, "2^n oscillators combined with 3-dB directional couplers for output power summing," *Proc. IEEE*, vol. 55, pp.2166-2167, Dec. 1967.
[3]M. Nakajima, "A proposed multistage microwave power combiner," *Proc. IEEE*, vol. 59, pp.24?-243, Feb. 1973.
[4]H. J. Kuno and D. L. English, "Millimeter-wave IMPATT power amplifier/combiner ," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp.758-767, Nov. 1976.
[5]G. P. Riblet, "Compact planar microstrip-slotline symmetrical ring eight-port comparator circuits," *IEEE Trans. Microwave Theory Tech.*, pp.1421-1426, vol. MTT-38, Oct. 1990.
[6]T. Kawai, K. Iio, I. Ohta and K. Kaneko, "A branch-line-type eight-port comparator circuit," 1991 *IEEE MTTS Dig.*, Z-6, pp.869-872, June 1991.
[7]I. Ohta, T. Kawai, S. Shimahashi and K. Iio, "A transmission-line-type eight-port hybrid," 1992 *IEEE MTTS Dig.*, B-6, pp.119-122, June 1992.
[8]G. P. Riblet, "Synthesis of 8 port comparator circuits using Dicke's methods," private message.